



# Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

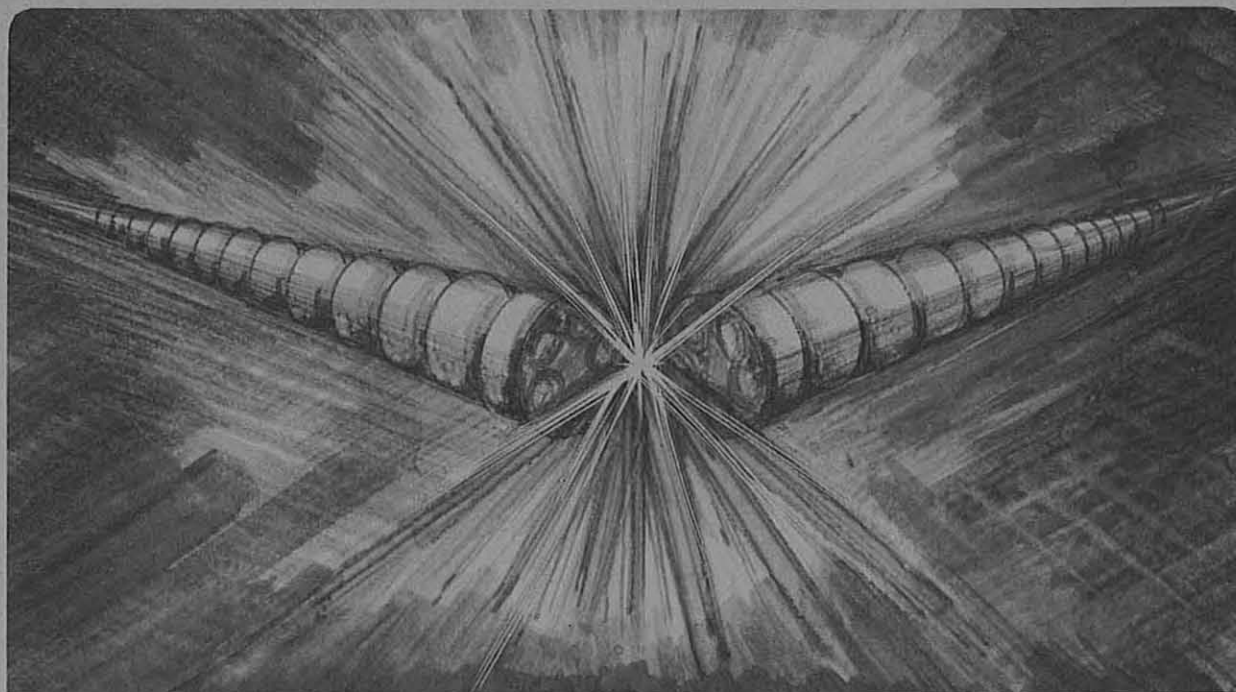
## Accelerator & Fusion Research Division

Presented at the Workshop on B-Factories,  
Stanford, CA, April 6–10, 1992,  
and to be published in the Proceedings

### **Bench-Marking Beam-Beam Simulations Using Coherent Quadrupole Effects**

S. Krishnagopal and Y.-H. Chin

June 1992



#### DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. Neither the United States Government nor any agency thereof, nor The Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or The Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or The Regents of the University of California and shall not be used for advertising or product endorsement purposes.

Lawrence Berkeley Laboratory is an equal opportunity employer.

# **Bench-Marking Beam-Beam Simulations Using Coherent Quadrupole Effects**

**S. Krishnagopal and Y.-H. Chin\***

*Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720*

June 1992

\*Work supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098

## Bench-Marking Beam-Beam Simulations Using Coherent Quadrupole Effects \*

S. Krishnagopal and Y.-H. Chin

Lawrence Berkeley Laboratory, University of California, Berkeley CA 94720

### ABSTRACT

Computer simulations are used extensively in the study of the beam-beam interaction. The proliferation of such codes raises the important question of their reliability, and motivates the development of a dependable set of bench-marks. We argue that rather than detailed quantitative comparisons, the ability of different codes to predict the same qualitative physics should be used as a criterion for such bench-marks. We use the striking phenomenon of coherent quadrupole oscillations as one such bench-mark, and demonstrate that our codes do indeed observe this behaviour. We also suggest some other tests that could be used as bench-marks.

### 1. INTRODUCTION

There is much experimental evidence to suggest that the beam-beam interaction is largely responsible for the observed saturation of the achieved tune-shift parameter in  $e^+e^-$  colliders. However, the dynamical reasons for this limitation are little understood. Because the force is strongly nonlinear the dynamics is complex, and theoretical analyses have limited predictive power. For this reason one often turns to computer simulations for help.

Even with simulations, it is not easy to model all the real-life effects that occur in operating storage-rings. Consequently approximations are called for, which, depending on the particular aspects of the beam-beam interaction being investigated, can vary widely. For example, when incoherent phenomena are thought to limit machine performance, 'weak-strong' simulations can be employed; the study of coherent phenomena requires 'strong-strong' simulations.

The plethora of experimental situations and design goals have led, inevitably, to a proliferation of beam-beam simulation codes that differ considerably in the scope and extent of the physics they purport to model. Unfortunately, this very variety makes it impossible to compare the codes directly against each

other. On the other hand, it is vital to correctly and exhaustively bench-mark these codes so that the results they spew out may be believed, and discrepancies with experimental data may be attributed to incomplete modelling of the physics, rather than to erroneous computer codes.

The ultimate test of the reliability of a beam-beam code would lie in its ability to reproduce at least some subset of experimental data gathered at an operating storage ring. At present, however, because of the many complicated effects that enter in real life (both dynamical and inadvertent – such as wrongly wound magnets!), that is not possible. Therefore, as a first step towards achieving the goal of reliably bench-marking beam-beam simulation codes, we suggest in this paper the use of *qualitative* physics in the effort. By this we refer to physical phenomena predicted by (admittedly simplified) theories, that in the appropriate parametric regimes have a clearly defined functional dependence. The task of the simulation would be to, in those regimes, reproduce the general features of the prediction.

We feel that this approach enables a more direct and unambiguous test of the correctness of a simulation code – as opposed to, for example, comparing the degree of beam blow-up under different operating conditions. Especially in the case of the beam-beam interaction, where the force is nonlinear and (generally) repeated, while theory cannot be very quantitative, qualitative predictions are more dependable.

\* Supported by the U.S. Department of Energy, Division of Nuclear and High Energy Physics, under contract No. DE-AC-03-SF-00098.



Further, these predictions are independent of the particular features of a simulation code. Sometimes the differences in these features, though slight and subtle, can yet lead to widely different results. Such details are not generally mentioned in reports, making comparisons difficult. For example, some codes employ the feature of 'exponential averaging', where the average value of the beam-size over a number of turns is used instead of the value at a particular turn, in order to increase numerical stability. While this is useful in studying incoherent phenomena, it has the very serious side-effect of suppressing certain kinds of coherent phenomena.

As an illustrative example of the kind of qualitative physics we are talking about, we consider in this paper the case of quadrupolar coherent beam-beam resonances. In the next section we discuss briefly the theory behind this phenomenon and, without going into derivations, sketch certain basic features of these resonances, as predicted by the theory. We also discuss some earlier simulation results that observe this behaviour. In Section 3 we show how two simulation codes, developed independently and for different purposes, successfully observe these distinctive features. In Section 4 we suggest some other tests that are based on different physics, and may be useful in bench-marking other features of beam-beam codes.

## 2. THEORETICAL PREDICTIONS

Coherent dipole motion is routinely observed in operating storage rings. Some of the experience with space-charge compensation at DCI also suggests that coherent effects may play an important role in beam-beam dynamics [1]. One general technique in the analysis of *quadrupole* coherent phenomena is the use of the Vlasov equation. This has been done for the beam-beam interaction by Chao and Ruth [2] and by Dikansky and Pestrikov [3]. They start with an equilibrium phase-space distribution  $f_0$  for the two beams. Assuming small harmonic perturbations  $\Delta f$  from the equilibrium distribution, the stability of these modes is analyzed using the linearized Vlasov equation. There are, of course, many approximations involved, besides the linearization of the Vlasov equation. Importantly, because the Vlasov equation is being used, effects of synchrotron radiation are not included; a significant drawback in modelling  $e^+e^-$  storage rings.

Broadly speaking, the results of this analysis are characterized by the appearance of even-order non-

linear coherent resonances. The widths of these resonances, i.e. the range of beam currents over which they occur, and the threshold for their onset, depend on the operating tune. The theoretical model is, of course, very much simplified. Nonetheless it makes certain specific predictions that one can attempt to verify using simulations.

In an attempt to make closer contact with reality, a multi-particle tracking program was developed to look at coherent behaviour [4]. It was strong-strong, and had the feature of being able to self-consistently calculate fields from non-Gaussian beam distributions. In addition, radiation damping and quantum excitation effects were included. Further details of the simulation can be obtained from Ref. 4; for our purposes we only note that using this program coherent beam-beam resonances that had the same general structure as predicted by the Vlasov model were observed. In particular, *at tunes just below the quarter-integer, the beams were observed to execute anti-correlated, period-2 motion*: on a given turn one beam was dense while the other was hollow; on the next turn the two beams exchange states, and this pattern repeated indefinitely. This behaviour is very striking in a plot of beam-size vs turn-number, as we shall see later. Further, a definite threshold was observed for the onset of the oscillations. (More recently beam-beam simulations that use macro-particles, developed at CEBAF, have also observed these resonances [5].)

These simulations extend the validity of the rather simple theoretical model into more realistic regimes, and suggest that this is a phenomenon which can occur in operating storage rings under the right (or wrong!) operating conditions. It then becomes important for a beam-beam simulation code that, for example, may be used in the design of a B-factory, to be able to predict this behaviour. We therefore use the observation of these period-2 beam-size oscillations as a criterion for our bench-mark test.

## 3. BENCH-MARKING THE SIMULATIONS

We now turn to two beam-beam simulation programs, which we shall call Simulation A and Simulation B, that we wish to bench-mark against the theoretical prediction of period-2 anti-correlated beam-size oscillations near the quarter-integer. The two simulations were developed quite independently and for different purposes. In order to bench-mark them, we constrained the simulations such that they were now modelling the same physics, and would there-

fore be expected to yield similar results.

Simulation A was developed by R.H.Siemann and S.Krishnagopal [4], with the original intent of looking at the consequences to coherent dynamics of being able to calculate fields from non-Gaussian beam distributions. The program is strong-strong and two-dimensional – there is no longitudinal dynamics. The beams are assumed to be symmetric. The arc transport is linear. The damping and excitation effects of radiation are put in once a turn and according to a scheme described in Ref. 6; this is different from the technique used by Simulation B. The beam-beam interaction is general and makes no assumption about the beam-distribution. The particles are cast onto a grid and their net electric field is calculated. There is only one collision per turn. The beams start out round, and though they are not constrained to remain so, the beam-beam kick calculation algorithm fails if the beam-profiles develop substantial eccentricity.

Simulation B was developed by K.Yokoya and modified by Y.-H. Chin, and in its present version is used in design studies for asymmetric B-Factories. This program is three-dimensional. The beam-beam interaction includes thick-lens effects by slicing the beam up into many (typically five) longitudinal chunks. The beam-beam kick calculation assumes a Gaussian beam-distribution, and uses the formula of Bassetti and Erskine [7]. Here too the lattice is assumed linear and radiation is put in only once a turn, though according to a scheme different from that used by Simulation A. There is no restriction on the beam-profile.

Table 1: Parameters used by Simulations A & B

Energy ( $E_0$ )	5.3 GeV
Revolution Period ( $T_0$ )	2.56 $\mu$ sec
Transverse Emittances ( $\epsilon_x = \epsilon_y$ )	$1 \times 10^{-7}$ m
Amplitude Functions ( $\beta_x^* = \beta_y^*$ )	3 cm
Betatron Tunes ( $Q_x = Q_y$ )	0.72
Damping Decrement ( $\delta$ )	$1 \times 10^{-3}$
Current ( $I$ )	35 mA
Nominal Beam-Beam Parameter ( $\xi_0$ )	0.1225
Number of test particles	1000

In order to make the two simulation programs model the same physics, they were stripped of many of their features and simplified. They were made: (1) symmetric, (2) two-dimensional (no longitudinal motion); (3) the beam-beam kick assumed Gaussian beam-distributions, (4) the beam-profiles were constrained to remain round, (5) coherent dipole motion was removed by setting the centroids of the beams to zero after each turn (thus simulating an idealized feedback system). Under these simplified conditions the two codes should now be modelling the same physics and, for the same input conditions, ought to give the same results. We picked the parameters shown in Table 1 for both programs; these correspond to a regime in which one expects to see the period-2 coherent oscillations described in the previous section. The two programs were allowed to run for  $2\frac{1}{2}$  damping times, and we then looked at the horizontal beam sizes over the next 25 turns.

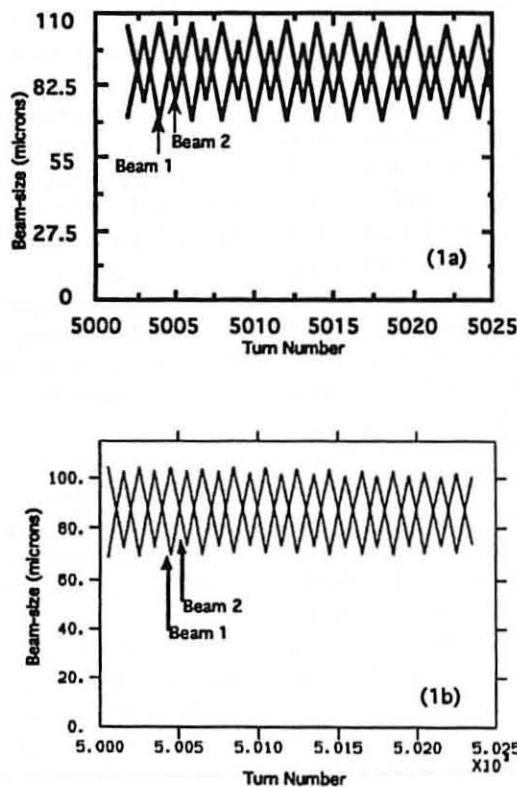


Figure 1: Results from (a) Simulation A, and (b) Simulation B. Horizontal beam-size  $\sigma$  as a function of turn-number for 25 turns after the first 5,000. The period-2, anti-correlated nature of the oscillations is evident. Parameters for the runs are from Table 1. The nominal size is  $55\mu$ m.

The results for Simulation A and B are shown in Figures (1a) and (1b) respectively. In both cases the period-2 nature of the beam-size oscillations, and their anti-correlated nature between the two beams, is unmistakable. In Simulation A the beam sizes oscillate between about  $72 - 105 \mu\text{m}$ , while in Simulation B the range is about  $70 - 100 \mu\text{m}$ . At the same tune-point both simulation programs were also used to find the current threshold for the onset of this instability. The threshold was found to be 23 and 24 mA respectively for Simulations A and B. In either case the agreement is good.

#### 4. OTHER TESTS

In the previous Section we have given an illustration of a useful benchmark that may be derived from the qualitative predictions of theory – in this case of period-2 coherent quadrupolar oscillations, predicted by the Vlasov equation models. It tests one particular aspect of a beam-beam code. We now list a few other theoretical results that could form the basis for developing other benchmark tests to test other aspects of a simulation code:

- (1) In the weak-strong approximation, when there is no longitudinal motion, the dynamics is dominated by betatron resonances of the form  $mQ_x + nQ_y = q$ . The strengths of these resonances can be calculated using a Hamiltonian analysis [8], and, for small amplitudes of the particles in the weak beam, they can be shown to vary as the  $J_0$  Bessel function. A simulation benchmark could be developed by looking at the functional dependence of the beam blow-up on particle amplitude; it should have the same Bessel function dependence.
- (2) If synchrotron oscillations are included (but the strong beam is assumed to have no longitudinal size), then synchro-betatron resonances make their appearance, and it can be shown [8,9] that the strength of the  $p$ th sideband varies, for small amplitudes, as the  $J_p$  Bessel function.
- (3) If the finite longitudinal extent of the beam is taken into account, so that the 'thick-lens' effect comes into play, then theory predicts [9] that resonance strengths will decrease as the bunch-length of the strong beam is increased, in approximately a Gaussian manner.
- (4) For strong-strong simulations, we have already considered coherent quadrupolar effects. Similarly, one can allow the beam centroids to move freely and then look at the threshold for the onset of coherent dipole motion.

#### 5. CONCLUSIONS

We have argued in Section 1 that physical phenomena or behaviour predicted by theory should be used to benchmark beam-beam simulation codes. As an example we considered coherent quadrupole oscillations in Section 2, and in Section 3 benchmarked two different simulation programs against this phenomenon. In Section 4 we outlined some other physical predictions that could form the basis for developing benchmarks that would test other aspects of a beam-beam simulation code.

We do not ask that a simulation program 'pass' or 'fail' based on these tests. There must, of course, be a more exhaustive set of benchmarks. Nor are we unmindful of the ultimate need for simulations to be able to explain experimental results. We have merely tried here to suggest one criterion on which the choice of such a set may be based, and we hope to have re-triggered a wider debate on the importance of developing benchmarks for beam-beam simulation codes, and on the means of achieving this goal.

#### REFERENCES:

- [1] J. Le Duff *et al.*, in *Proceedings of the Eleventh International Conference on High-Energy Accelerators, Geneva, 1989*, edited by W.S. Newnam (Birkhauser, Geneva, 1980), p.707.
- [2] A.W. Chao and R.D. Ruth, *Part. Accel.* **16**, 201 (1985).
- [3] N.S. Dikansky and D.V. Pestrikov, *Part. Accel.* **12**, 27 (1982).
- [4] S. Krishnagopal and R.H. Siemann, *Phys. Rev. Lett.* **67**, 2461 (1991).
- [5] Rui Li, private communication.
- [6] S. Krishnagopal and R.H. Siemann, Cornell Internal Note CBN/88-8, 1988; R.H. Siemann, CBN/89-2, 1989 (unpublished).
- [7] M. Bassetti and G.A. Erskine, Report No. CERN-ISR-TH/80-06, 1980 (unpublished).
- [8] F.M. Izrailev and I.B. Vasserman, in *Proceedings of the 7th All-Union Conference on Charged Particle Accelerators*, Dubna, USSR 1981 (Joint Institute for Nuclear Research, Dubna, 1980), p.288.
- [9] S. Krishnagopal and R.H. Siemann, *Phys. Rev. D* **41**, 2312 (1990).